Reformulation of the polarized radiative transfer problem with the Magnus expansion

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Abstract

The light emitted by the sun is polarized in hundreds of spectral lines by Rayleigh scattering occurring between photons and ions, which produces spectropolarimetric signals that are sensitive to the physical state of the stellar plasma. Polarization is indeed the only way of measuring solar and stellar magnetic fields, being thus key to understand the solar atmosphere. However, in order to use polarization for astrophysical diagnosis one has to solve the radiative transfer equation (RTE) for polarized light. Until now, this has been done inaccurately and locally by assuming that the system propagation matrix is constant in the integration domain. In this contribution, we shall present a new formalism that reformulates this problem and lays the foundations for a non-local geometrical integration of the RTE in the Lorentz/Poincaré group of rotations without assuming constant properties. To this aim, we take advantage of the suitability of the Magnus expansion for obtaining increasingly accurate solutions and combine it with a detailed algebraic characterization of the propagation matrix involving a new generalized Lorentz matrix. Thus, we reformulate the homogeneous and the inhomogeneous solution to the RTE and obtain a compact analytical expression of the evolution operator that supports arbitrary spatial variations to first order in the Magnus expansion, paving the road to higher orders. This gives the first non-local formal solution to the RTE that furthermore is efficient, consistent, natively accurate (preserves group structure of the solution), and that separates the integration part from the formal part. Our formulation leads to a new family of numerical methods for the RTE, suggests innovative ways of accelerating polarization syntheses and inversions in solar physics, and is valid for other universal physical problems sharing the Lorentz/Poincaré algebra of special relativity. Besides, it motivates some exciting questions: what is the relation between the Magnus expansion and fractality? and with the Planck's law for black-body radiation? Is it possible to represent the full Magnus expansion as a matricial function of the propagation matrix?

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