Time-Varying Expected Returns, Conditional Skewness and Bitcoin Return Predictability

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1 Introduction

Considerable evidence shows that the distributions of asset returns cannot be adequately characterized using only the mean and variance (Kraus and Litzenberger, 1976; Leland, 1997; Harvey and Siddique, 2000; Landsman et al., 2020). This fact brings us to the third moment of the asset distribution, skewness. Following Jondeau et al. (2019), the three-moment CAPM implies that market skewness should be a predictor of market returns. In fact, according to Jurczenko and Maillet (2011), skewness can capture some of the factors used in the extended CAPM, such as industry, book-to-market value, size or momentum effects. Moreover, when dealing with the theoretical foundations of rational choices, a mean-variance-skewness criterion can be justified (Arditti, 1967; Prakash et al., 1996; Kane, 1982), and risky behavior by a risk-averse agent can be explained (Prakash et al., 1996; Golec and Tamarkin, 1998).

Based on these findings, during recent years, several papers have analyzed the capacity of different measures of skewness to predict subsequent asset returns. Holding everything else constant, investors should prefer assets with positive skewness (Scott and Horvath, 1980; Harvey and Siddique, 2000). Therefore, assets with positive (negative) skewness are more (less) attractive to investors and consequently have lower (higher) expected returns in the next period. Thus, a negative relationship should exist between asset skewness and its expected return in the next period.

At the individual asset level, some evidence shows the ability of individual skewness to predict future individual asset returns (Boyer et al., 2010; Conrad et al., 2013; Amaya et al., 2015). However, much less clear evidence exists at the aggregate market level, where a clear relationship between market skewness and future market returns has not been found (Chang et al., 2011; Jondeau et al., 2019), although average sample skewness in individual stocks is a good

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predictor of future market returns with the expected (negative) sign. Following this line of research, Serna (2023), in a study conducted for several stock indices and exchange rate series that capture recent major market crises, proposes replacing the sample estimators of skewness, which respond too slowly to changes in market conditions, with conditional estimators of skewness and finds that conditional skewness estimated from a GARCH-type model outperforms sample skewness in predicting future market returns with an unexpected (positive) sign. The rationale behind this surprising finding is that during major crises, where substantial declines in asymmetry occur, investors can find good investment opportunities at attractive prices, increasing demand and therefore reducing the return in the following period, creating a positive relationship between skewness and future asset returns.

Focusing on the literature on Bitcoin, Jia et al. (2021) document strong evidence that sample skewness is negatively related to future Bitcoin returns, using data up to 2019. Ahmed and Mafrachi (2021) find that both sample skewness and hyperskewness show a significant ability to predict subsequent Bitcoin returns. Important to note is that skewness in these two articles has been estimated on a sample basis. In this study, we follow this line of research by analyzing the conditional skewness obtained from a GARCH-type model compared to sample skewness and sample and conditional volatility in predicting future Bitcoin returns using a more recent time series up to 2022, which in the cryptocurrency market is a period of relative stability followed by a recent major period of turmoil characterized by high volatility and major cracks. Interestingly, during the first period (2018–2020), characterized by relative stability, the relationship between conditional skewness and future Bitcoin returns has the expected (negative) sign. However, during the second period (2021–2022), characterized by major turmoil, the relationship is positive, confirming the results in Serna (2023) for stock indices and exchange rate series.

Based on these findings, a dynamic buy and sell strategy of buying or selling Bitcoin based on the estimated conditional skewness is proposed. This dynamic strategy is profitable in many cases and outperforms a static buy and hold strategy. Notably, these results do not necessarily contradict the efficient market hypothesis. In fact, although many papers have identified some degree of predictability in asset returns (Liu et al., 2019; Levich et al., 2019), Campbell et al. (1997) argue that some degree of predictability may be due to frictions in the trading process, to market microstructures and—above all—to the variability in the expected returns caused by changes in market conditions. Therefore, the fact that asset returns present a certain level of predictability can be interpreted as the premium that investors demand in exchange for facing these dynamic risks. In the context of this article, the capacity of conditional skewness to predict future Bitcoin returns and the profitability of the dynamic buy and sell strategy based on this predictive power can be viewed as the reward that investors demand for bearing the risk associated with changing conditions in the cryptocurrency market that generate time-varying expected returns, as probably occurred during the second part of the time series analyzed (2021–2022), which was characterized by major turmoil.

2 Methods

The data series employed in this paper is composed of Bitcoin daily closing prices, expressed in USD, from 7/1/2017 to 12/6/2022 (1985 daily observations). Figure 1 depicts the time series evolution of Bitcoin daily prices during the period considered. Quite striking to observe is that until 2020, the Bitcoin market shows a relatively stable evolution. However, from 2021 onwards, the Bitcoin market seems to have entered a turbulent period with greater volatility and large declines. As explained below, sample and conditional variance and skewness are estimated in a rolling window containing 200 previous daily Bitcoin returns, and the out-of-sample period begins on 1/18/2018.

To confirm whether the series of daily log returns in the two subperiods, the first period characterized by relative stability (1/18/2018–12/31/2020) and the second one is characterized by major turmoil (1/1/2021–12/6/2022), are from the same continuous (H0) or different (H1) distributions, a two-sample Kolmogorov–Smirnov test is performed. The test rejects the null hypothesis at the 5% significance level (p value equal to 0.0119). Therefore, the previous visual analysis that suggested that the Bitcoin market shows two different subperiods—one of relative calm followed by a period of greater turbulence—is confirmed by the Kolmogorov–Smirnov test.

Each day sample and conditional variance and skewness are estimated in a rolling window containing the 200 previous daily log returns. Let r_t be the Bitcoin log-return on day t. The sample variance (h_t^s) and skewness (sk_t^s) on day t can be computed as:

$$h_t^s = \frac{1}{200}\sum_{i=t-1}^{t-200}(r_i - \bar{r}_t)^2, \quad sk_t^s = \frac{1}{200}\sum_{i=t-1}^{t-200}\left(\frac{r_i - \bar{r}_i}{\sigma_t}\right)^3, \tag{1}$$

where $\bar{r}_t$ and σ_t are the average Bitcoin return and volatility on day t, which are calculated as the sample average and the sample standard deviation of the previous 200 daily returns, respectively. Using this procedure, 1784 daily estimations of sample variance and skewness are obtained from 1/18/2018 to 12/6/2022.

Conditional estimations of variance and skewness are obtained by fitting the GARCHS model employed by Serna (2023), which is a reduced version of the GARCHSK model by León et al. (2005)[1]:

[1] GARCHSK stands for Generalized Autoregressive Skewness and Kurtosis, whereas GARCHS is a reduced version of the GARCHSK model with constant kurtosis and no structure in the mean equation. In the estimates carried out the coefficients of the ARMA structure were barely significant and to reduce the number of parameters to be estimated (which is important in the estimation process) it was decided to propose a model with no structure in the mean equation.

$$
\begin{gathered}
r_t = \varepsilon_t;\ \varepsilon_t | I_{t-1} \sim GT(0, h_t^c);\ \eta_t = \varepsilon_t h_t^{c\frac{1}{2}}, \\
h_t^c = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 h_{t-1}^c, \\
sk_t^c = \gamma_0 + \gamma_1 \eta_{t-1}^3 + \gamma_2 sk_{t-1}^c,
\end{gathered} \tag{2}
$$

where I_{t-1} is an information set containing the information available at time $t-1$, GT is the transformation proposed by Gallant and Tauchen (1989) of the Gram-Charlier series expansion of the normal density function truncated at the fourth moment, $h_t^c = var(r_t|I_{t-1})$ and $sk_t^c = skewness(\eta_t|I_{t-1})$. The model parameters β_0, β_1, β_2, γ_0, γ_1 and γ_2 are estimated using the maximum likelihood method.[2]

3 Results

In this section, we analyze whether the estimations of the sample and conditional variance and skewness obtained in Section 2 are able to predict one-day-ahead Bitcoin returns. To do this, we use the simple regression procedure proposed by Jondeau et al. (2019):

$$
r_t = \alpha_0 + \alpha_1 \cdot h_t^c + \alpha_2 \cdot sk_t^c + \alpha_3 \cdot h_t^c + \alpha_4 \cdot sk_t^c + e_t. \tag{3}
$$

As explained in Section 2, in both cases (sample or conditional), the variance and skewness estimations are obtained using the previous 200 daily Bitcoin log-returns in a rolling window from $t-1$ to $t-200$.

The estimation results of Model (3) are presented in Table 1 for the whole out-of-sample period (1/18/2018–12/6/2022), the first subperiod (1/18/2022–12/31/2020) and the second subperiod (1/1/2021–12/6/2022). In all cases, the p values shown in the table are obtained based on the Newey–West heteroskedasticity and autocorrelation-consistent standard errors. If we consider the whole out-of-sample period, only the conditional variance is weakly significant (at the 10% level) in predicting one-day-ahead Bitcoin returns. However, the results obtained from using the estimation model in (3) in the two subperiods considered are different. Specifically, in the first subperiod—a period of relative calm in the Bitcoin market as explained in Section 2—although the sample moments are weakly significant (at the 10% level), the most significant variable in the regression is the conditional skewness (at the 5% level). Furthermore, the conditional skewness is a predictor of future market returns with the expected (negative) sign (Jondeau et al., 2019).

As in the first period, in the second one—a period of major turmoil in the Bitcoin market as explained in Section 2— the most significant variable in the regression model (at the 5% level) is conditional skewness. However, in this case, a positive relationship exists between conditional skewness and one-day-ahead Bitcoin returns. This result is in line with the findings of Serna (2023), who found a positive relationship between conditional skewness and future market returns using stock indices and exchange rate series in a recent period of major turmoil.

[2] See León et al. (2005) for details about the log-likelihood function.

As mentioned in the Introduction, this result can be explained by the fact that investors can find prices attractive and increase buying pressure when a large market crash occurs that sharply reduces skewness, thus reducing the return in the next period.

As discussed in Section 2, the Bitcoin market is characterized by a high degree of variability, with expected returns changing rapidly from one period to another. As in Campbell et al. (1997), time-varying expected returns can generate a certain degree of predictability in asset returns. This fact can explain the predictive capacity of conditional skewness found in this paper and that at first may seem to contradict the efficient market hypothesis.

Once the predictive capacity of the conditional skewness to predict future Bitcoin returns has been verified, a natural step is to propose an investment strategy to attempt to take advantage from this predictive capacity. Specifically, during the first (not turmoil) subperiod (1/18/2022–12/31/2020), a simple dynamic self-financing strategy is proposed to profit from the negative relationship between conditional skewness and future Bitcoin returns by buying (selling) Bitcoin every day whenever the one-day-ahead prediction of conditional skewness increases (decreases) with respect to the previous day. The buying (selling) pressure increases (reduces) the price in the next period, thus reducing (increasing) the return in the next period. During the second turmoil subsample (2021–2022), Bitcoin is bought (sold) every day whenever the estimated conditional skewness decreases (increases) with respect to the previous day. In all cases, the transaction costs are taken into account (next block fee in dollars per transaction). This is a self-financing strategy because every day, the total cumulative profit from the strategy is invested or financed until the following day at the Secured Overnight Financing Rate.[3]

In the first subperiod (2018–2020), from a zero investment, the cumulative profit obtained at the end of the period is 155,383.45 dollars. During the same period, the profit from a static buy and hold strategy is 16,454.61 dollars. However, in the second subperiod (2021–2022), from a zero investment, the cumulative profit at the end of the period is 109,761.72 dollars, whereas the profit from a static buy and hold strategy during the same period is -14,999.33 dollars. Therefore, although the dynamic buy and sell strategy based on one-day-ahead predictions of conditional skewness outperforms the simple buy and hold strategy in both subperiods, its reliability is much lower in the second subperiod.

These results suggest that although the conditional skewness shows significant predictive ability to predict one-day-ahead Bitcoin returns, it is not clear that this ability can be used to profit in a systematic way; thus, clear evidence against the efficient market hypothesis is not obtained. When implementing a dynamic buy and sell strategy, in periods of relative calm in the Bitcoin market—during which conditional skewness and future returns have a negative relationship—the predictive ability of conditional skewness seems to behave better than in

[3] Secured Overnight Financing Rate data are obtained from https://www.newyorkfed.org/markets/reference-rates/sofr

periods of major turmoil—during which conditional skewness and future returns have a positive relationship.

4 Conclusions

In this paper, we analyze the predictive ability of conditional skewness—estimated from a GARCH-type model for conditional variance and skewness—for predicting future Bitcoin returns. Conditional skewness outperforms sample skewness and sample and conditional variance in predicting one-day-ahead Bitcoin returns. Moreover, the relationship between conditional skewness and future Bitcoin returns is negative in periods of relative calm in the Bitcoin market and positive in periods of major turmoil.

Based on these results, a dynamic strategy for buying or selling Bitcoin every day based on the variation in the conditional skewness with respect to the previous day is proposed. This dynamic strategy, which takes into account transaction costs, outperforms a static buy and sell strategy. However, in the Bitcoin market, the profitability of the strategy is much clearer in periods of relative calm than in periods of major turmoil.

The predictive ability of conditional skewness to predict future Bitcoin returns and the profitability of the dynamic strategy based on the estimated conditional skewness can be associated with the variations in the Bitcoin market conditions that generate time-varying expected returns. In fact, time-varying expected returns seem to be a characteristic in the Bitcoin market. Following the extant literature (Campbell et al., 1997), the profitability of the strategy can be viewed as the reward that investors demand for bearing the risk associated with time-varying expected returns.

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Figure 1. Time Evolution of Bitcoin Prices

This figure shows the time series evolution of daily Bitcoin price during the period from 7/1/2017 to 12/6/2022.

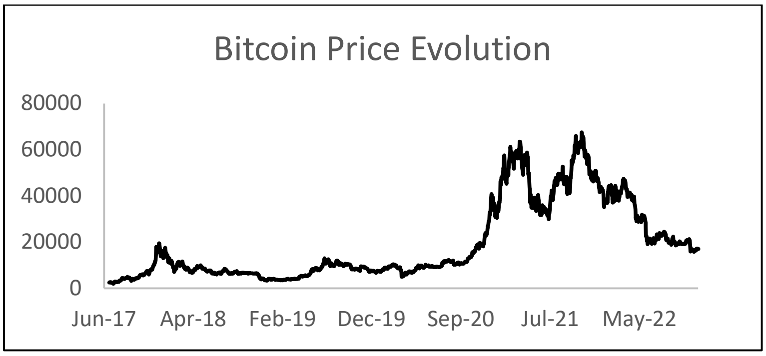

Table 1. Predictive Regressions for Bitcoin Returns

Variable	**Estimated coefficient (p-value)**		
	1/18/2018-12/6/2022 (1784 obs.)	**1/18/2018-12/31/2020** (1080 obs.)	**1/1/2021-12/6/2022** (704 obs.)
Constant (a)	0.0019	0.0036	-0.0063
	(0.2424)	(0.1325)	(0.02051)
h_t^c	0.1831*	0.1231	3.0466*
	(0.0803)	(0.1590)	(0.0620)
sk_t^c	-0.0022	**-0.0043****	**0.070****
	(0.1817)	(0.0314)	(0.0287)
h_t^s	-1.6129	-2.2625*	03737
	(0.1473)	(0.0879)	(0.4678)
sk_t^s	-0.0009*	-0.0010*	-0.0035
	(0.0591)	(0.0722)	(0.2218)
Adj. R^2	0.4%	0.91%	0.56%

This table shows the estimated parameters of the one-day-ahead regressions of the Bitcoin returns on the conditional and sample estimations of the variance and skewness for the whole sample period (1/18/2018–12/6/2022), the first subperiod (1/18/2022–12/31/2020) and the second subperiod (1/1/2021–12/6/2022). The p values shown in parentheses are calculated based on Newey–West heteroskedasticity and autocorrelation-consistent standard errors. The estimated values are reported, where * denotes significance at the 10% level, ** denotes significance at the 5% level (in bold), and *** denotes significance at the 1% level.